Multilateration system using glass spheres to determine station positions at better than 50 µm

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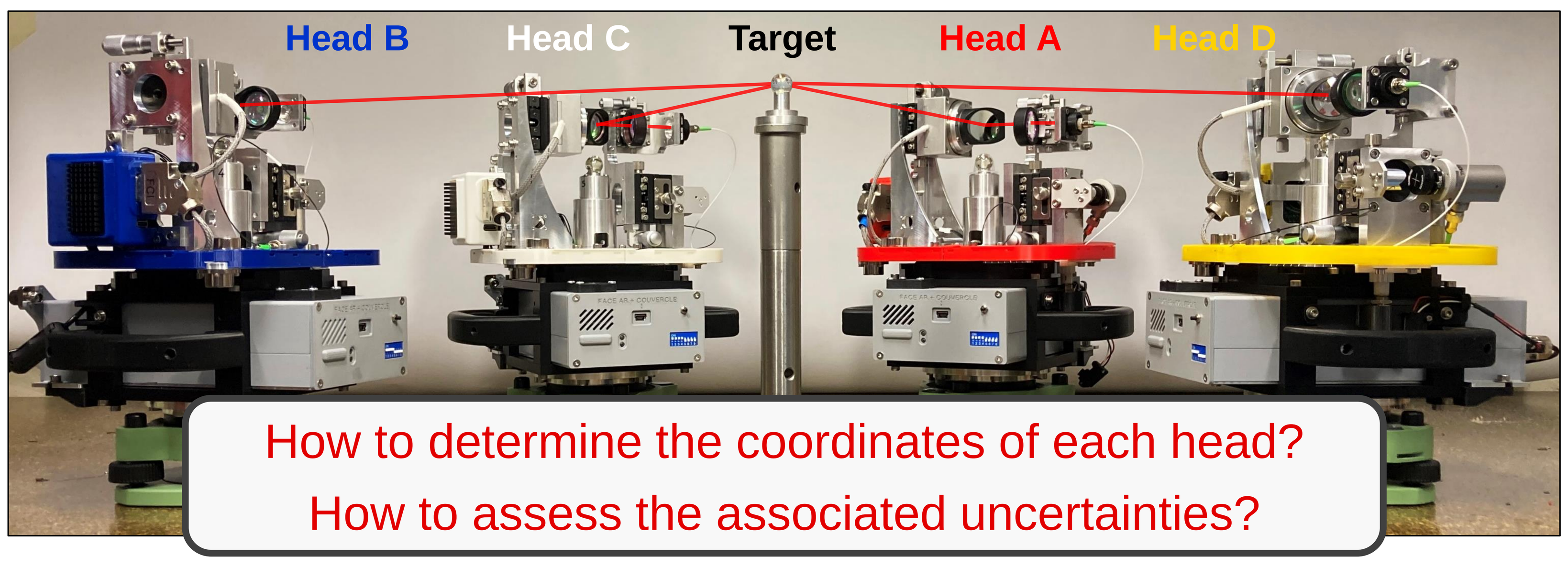
1. Problematic

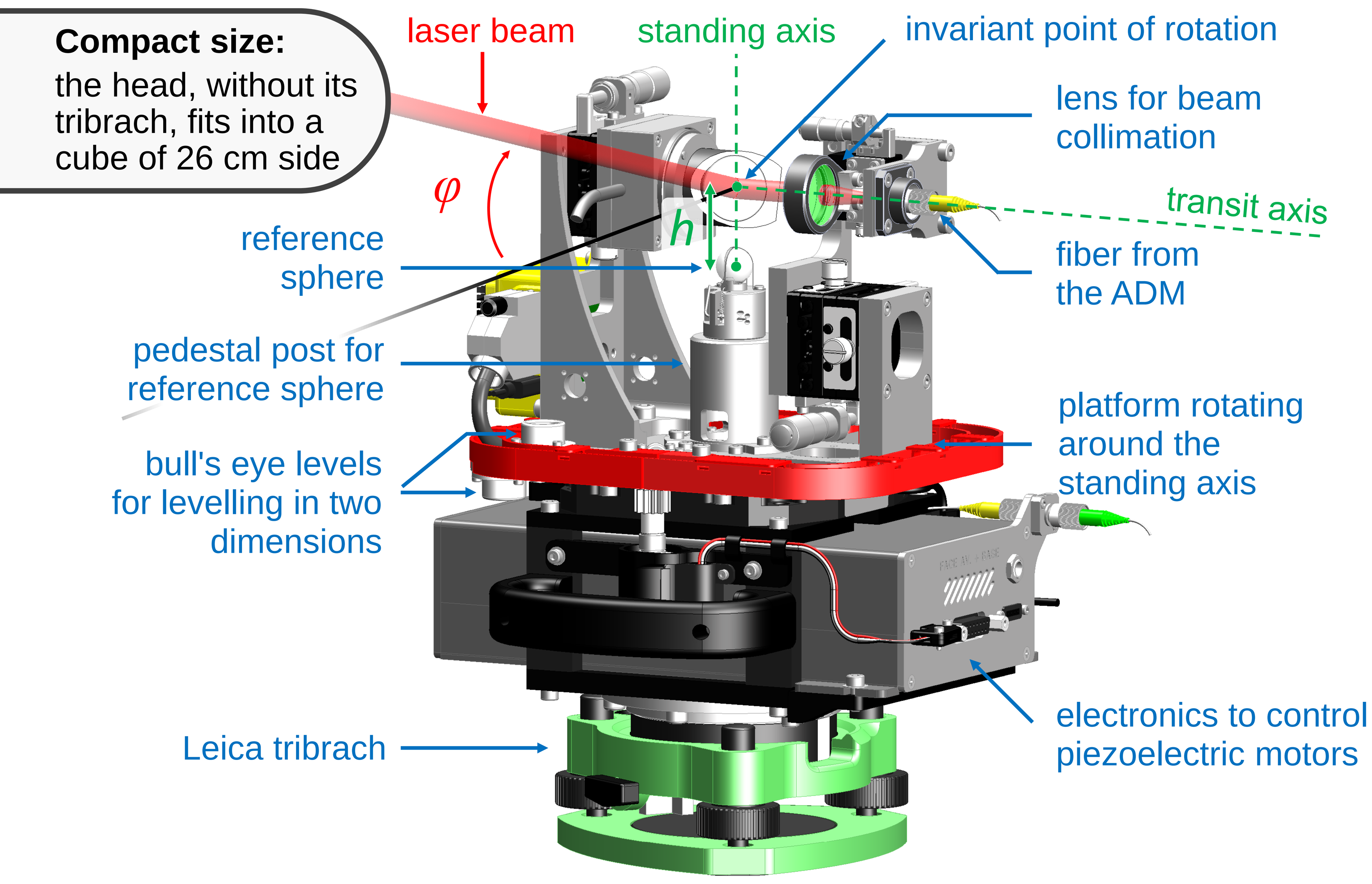
In classical multilateration technique, the coordinates of targets can be determined using distance measurements from stations (Total Stations, Laser Trackers, etc.), at least 4, whose positions are known. In our previous developments, we preferred to use multilateration technique with self-calibration because knowledge of the station coordinates is not required, but a large number of targets (at least 6) must be measured before to be able to determine both station and target positions. To switch to classical multilateration, and so real-time measurements, station positions must be determined.



2. Solution

We propose to determine all the distances between the measurement heads, using the heads themselves. This involves a new design of the heads with a built-in target: as retroreflector, we chose a glass sphere of refractive index $n = 2$, which is light, compact and visible from all directions.

This target was not placed at the invariant point of rotation, i.e. at the reference position of each head, because the laser beam already passes through it. It was placed on the standing axis, at $h \simeq 30$ mm below the transit axis, on a pedestal post.



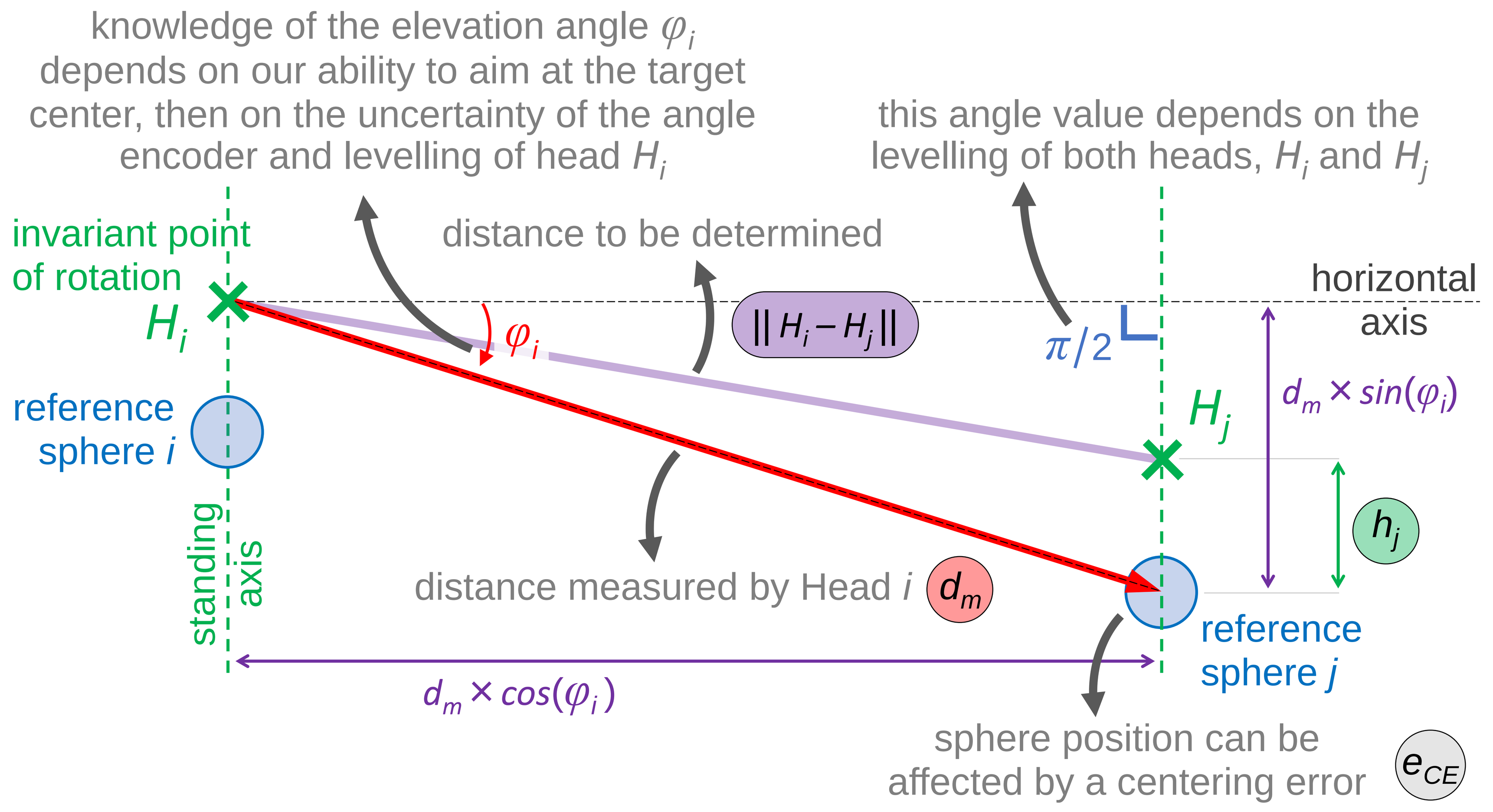
The input data are 6 inter-head distances, but also the 3 coordinates of one of the heads to define the origin of the reference frame, 2 coordinates of a second head to define the x axis, and 1 coordinate of a third head to define the xOy plane. This leads to the determination of 6 coordinates, i.e. the unknown variables, for 6 independent equations. This is done using the least squares method.

3. Inter-head distance $\| H_i - H_j \|$

The distances d_m and h_j are obtained thanks to the ADM developed in-house: in the first case, the head H_i aims at the reference sphere of the head H_j, while in the second case, the head H_j aims at its own reference sphere (φ_j = -90°). The distances are measured with about 5 µm uncertainty (k=1) when the air refractive index is carefully determined.

The angle φ_i is obtained from an angle encoder of ±0.017% uncertainty (k=1).

$$\boxed{\left\|H_i - H_j\right\|^2 = {d_m}^2 + {h_j}^2 - 2\, d_m\, h_j \times \cos\left(\frac{\pi}{2} - \varphi_i\right)}$$

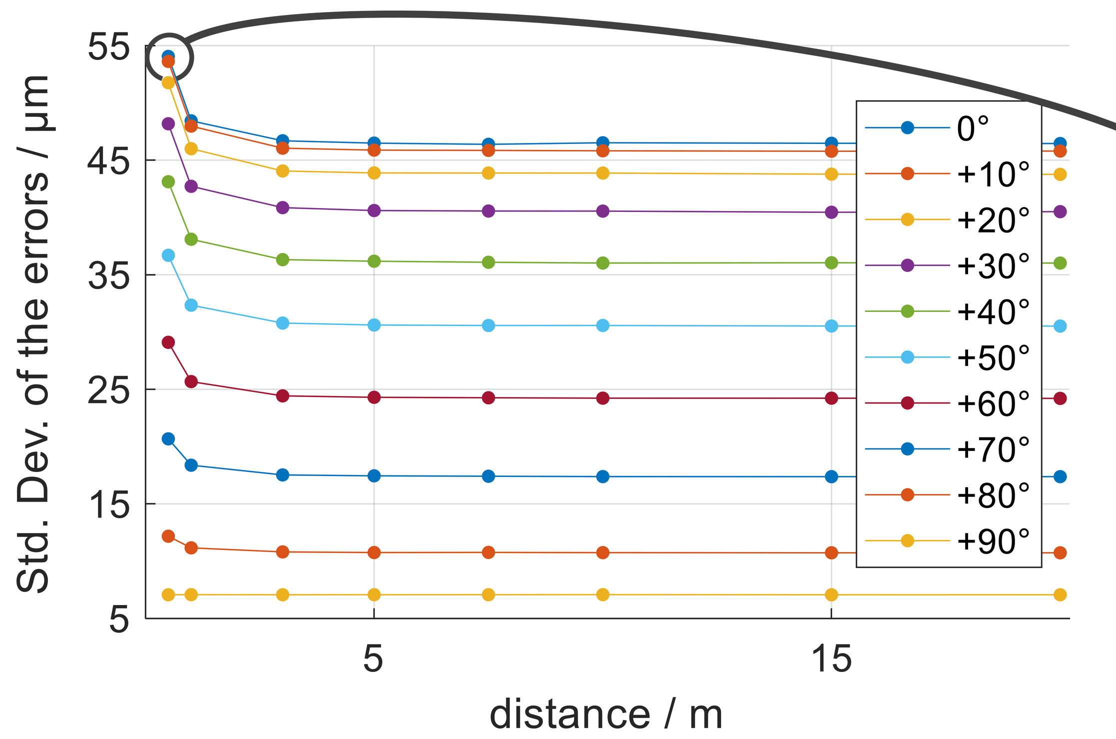


Levelling: when the level bubble is inside the circle, the levelling error follows a uniform distribution of 1 mrad uncertainty (k=1).

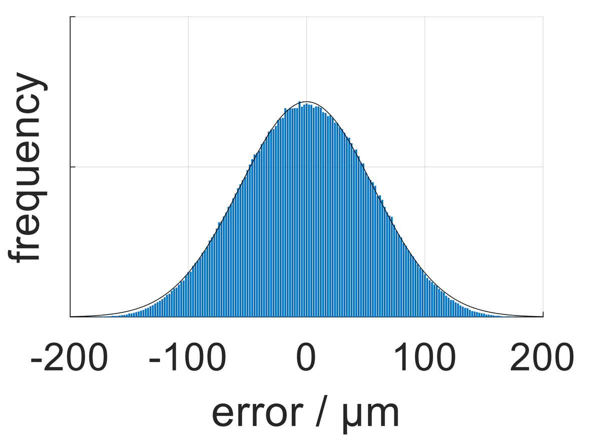
Aiming: it is optimized by the power received from the distant sphere. If the laser beam deviates from the target centre by ±0.8 mm, then the received RF power is typically halved (-3 dB), which is very perceptible by the ADM. Assuming a uniform distribution, a 0.5 mm uncertainty is considered, i.e. 0.5 mrad at 1 m or 0.05 mrad at 10 m (k=1).

Centering: the deviation e_{CE} of the reference sphere from the standing axis is treated as a Gaussian-distributed random variable of 20 µm uncertainty (k=1).

In the end, the uncertainty of the inter-head distance $\| H_i - H_j \|$ was determined by a Monte-Carlo simulation: 1 million of random variables were generated for e_{CE}, d_m, h, $\pi/2$ and φ_i. The result mainly depends on the elevation angle φ_i.

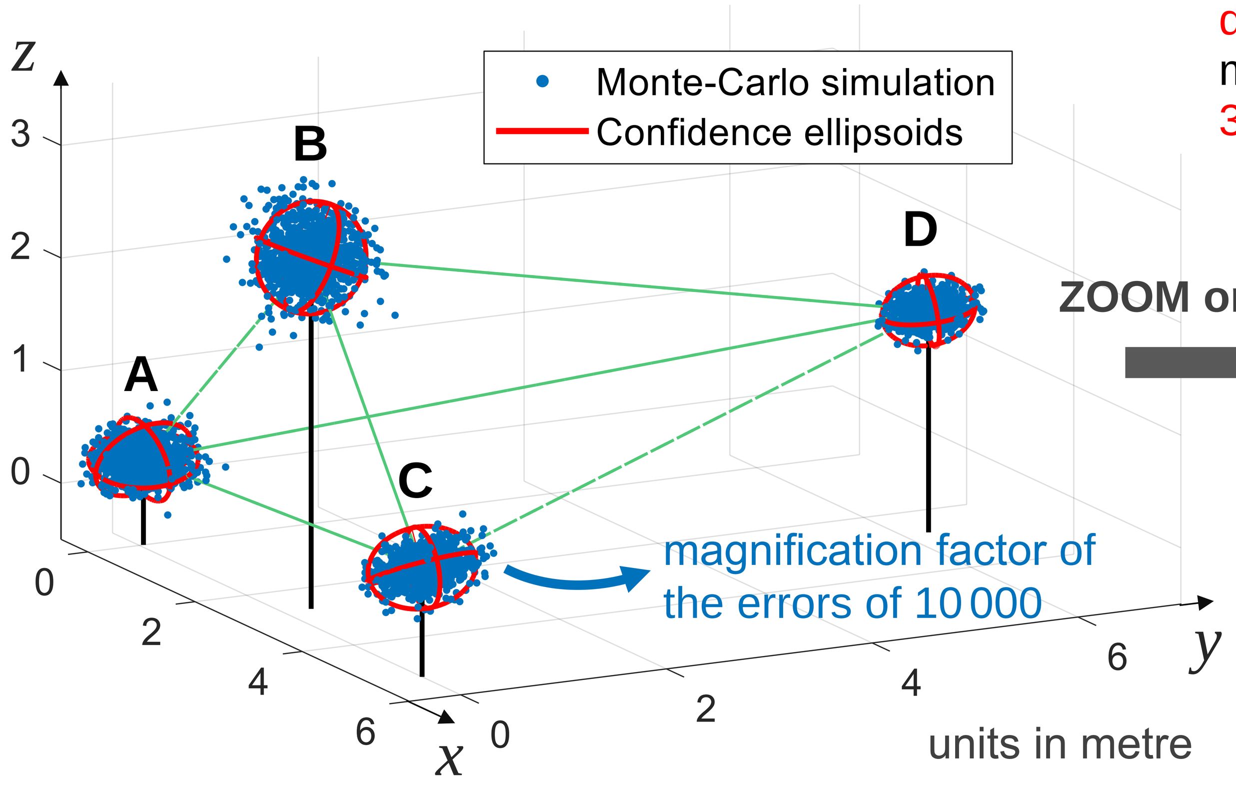


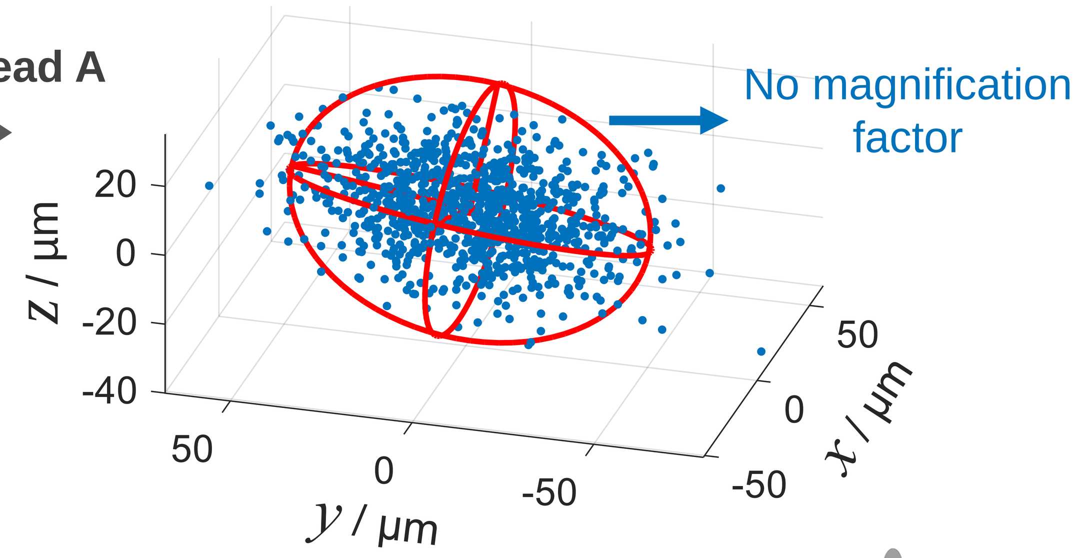
error distributions are gaussian ones. For the case of d_m = 0.5 m and φ_i = 0°:



4. From inter-head distances to head positions

The result depends on the geometrical arrangement used. As an example, we have chosen the one depicted below:





This arrangement corresponds to a real experiment from our previous works on multilateration. The inter-head distances range from 3.3 m to 6.9 m, and if they had been measured by our ADM, uncertainties would be between 35 µm and 47 µm (k=1) depending on the elevation angle.

1000 data sets of inter-head distance measurements were randomly generated. In this Monte-Carlo simulation, it was assumed that each distance was measured twice, from head H_i to head H_j, and vice versa. Then, for each data set, the head positions were determined by the least-squares method, and the resulting error with the true coordinates was calculated after a registration step. The result was 4 error matrices of size 3 × 1000, 1 per head, each with 3 coordinates.

The confidence ellipsoids of the estimated head positions were obtained by singular value decomposition (SVD). They are depicted in red on the left. Their axis half-lengths range from 32 µm to 54 µm (confidence regions of probability of 68%).

$\boxed{u(\text{position}) < 50\ \mu\text{m}\ (k=1)}$

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