

# Geometric partitioning by holistic approximation

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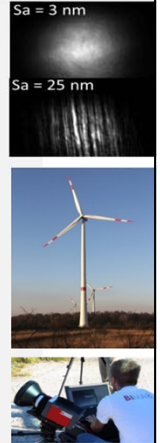
Bremen Institute for  
Metrology, Automation  
and Quality Science

# Measurement System Engineering

Research  
Teaching  
Knowledge

## Metrology

Automation      Quality control



$$\sigma_x \cdot \sigma_p \geq \frac{h}{4\pi}$$
$$\text{Var}(\hat{\theta}) \geq \text{CRB}(\theta)$$

### Methods

#### Measurement system theory

- Modeling and simulation
- Uncertainty relations
- Limits of measurability

#### Measurement system technique

- Optical high-speed systems
- Multi-sensor-systems
- Large volume metrology

→ model-based, dynamic measurement systems

### Applications

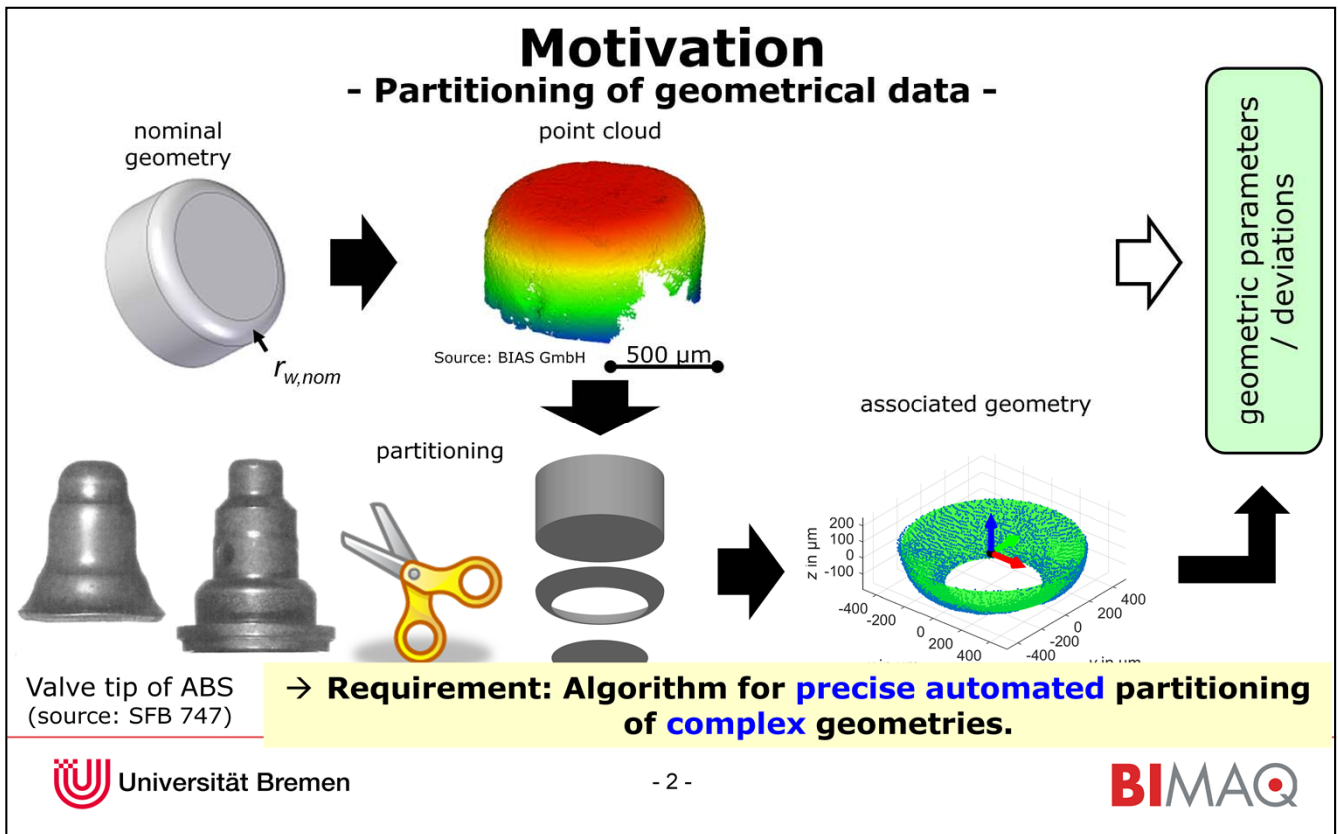
#### Production engineering & Material science

- Geometry and roughness metrology
- Thermography, Boundary layer analysis
- Optical in-process metrology & control

#### Wind energy systems & Flow processes

- Gear-wheel metrology
- Gear metrology
- Flow metrology

Bremen Institute for Metrology, Automation and Quality Science (BIMAQ),  
University of Bremen:  
Research fields.



Workpiece: drawn micro cup ( $D < 1 \text{ mm}$ ).

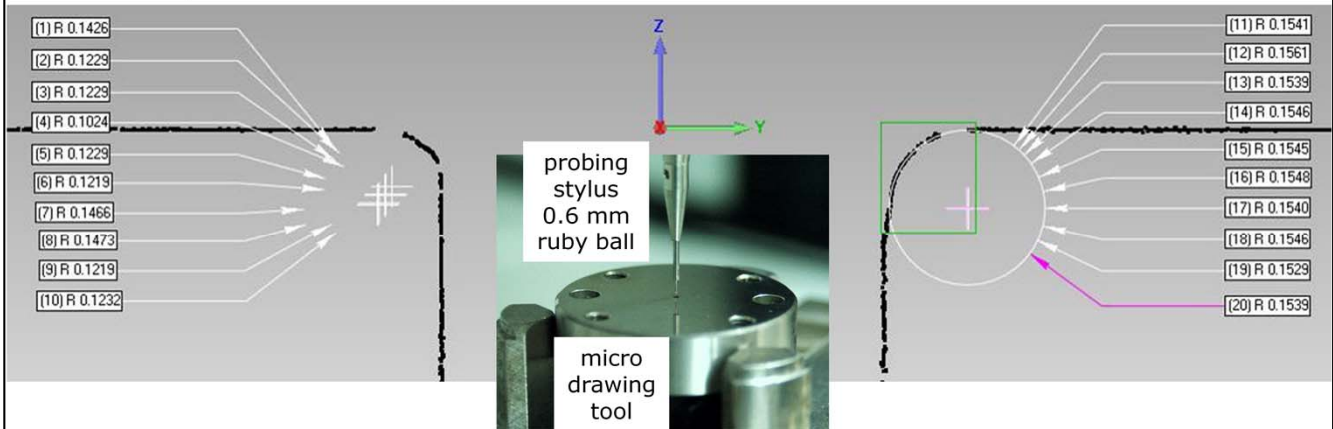
Cold formed micro workpieces are contained in numerous applications, e.g. valves of antilock braking systems (ABS)

Partitioning is one of 4 major operations within ISO GPS.

# Motivation

## - Manual partitioning -

- Example: Micro drawing tool



span: 53.7  $\mu\text{m}$  (30 % of nominal datum)

→ no automation  
→ poor reproducibility

# State of the art

## - Geometric partitioning / segmentation -

### Precise automated evaluation of complex dimensional data

	Metrology software	Edge detection	Attribute clustering	Region growing	Holistic Approximation (HA)
precise	-/+	-	o	-	+
automated	o	+	+	+	+
complex geometry	-	+	+	+	?
restrictions		sensitive to noise	sensitive to noise	over-/under-segmentation	<b>combination of simple geometries</b>

→ **Approach: Holistic approximation (HA) extended with root point iteration**

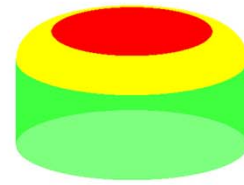
Holistic approximation:

- converges reliably
- tested for micro features
- outlier detection possible

# Outline

## Holistic approximation

- Method
- Results
  - a) Micro cup (3D)
  - b) Drawing die (ellipse)
  - c) Gear / involute
- Summary & outlook



# Method

## - Approximation approach -

### Holistic approximation

#### I. Geometric model

→ transformation parameters:  $\vec{a}_p$   
→ shape parameters:  $\vec{a}_g$  }  $\rightarrow d_i \leftarrow \vec{x}_i$  ( $i^{\text{th}}$  measuring point)

#### II. Separation rules

→ point associations with variable numbers of points:  $\vec{n} = [n_A, n_B, n_C, \dots]$

#### III. Least squares approximation:

$$\min_{\vec{a}_p, \vec{a}_g, \vec{n}} \left[ \sum_{i=1}^{n_A} (d_{A,i}(\vec{a}_p, \vec{a}_g))^2 + \sum_{j=1}^{n_B} (d_{B,j}(\vec{a}_p, \vec{a}_g))^2 + \sum_{k=1}^{n_C} (d_{C,k}(\vec{a}_p, \vec{a}_g))^2 + \dots \right]^{\frac{1}{2}}$$

→ **Automated combination of partitioning and approximation.**

### Characteristics:

- Exchange of norm possible (e.g. Chebyshev for shape deviations)
- Combination with statistic approaches for automated outlier detection

# Results

## - a) Micro cup (plane, cylinder and torus) -

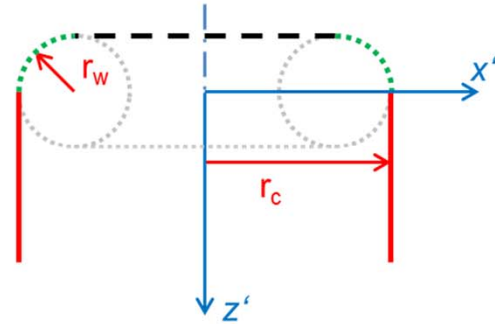
### Geometric model



transformation

$$\vec{x} \rightarrow \vec{x}'$$

- plane
- ... torus
- cylinder



transformation parameters:

$$\vec{a}_p = [\Delta x, \Delta y, \Delta z, \varphi_x, \varphi_y]$$

shape parameters:

$$\vec{a}_g = [r_c, r_w]$$



Parameter extraction for  
dimensional inspection

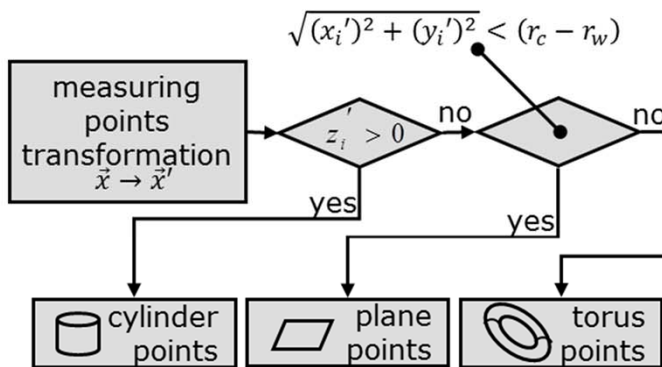


# Results

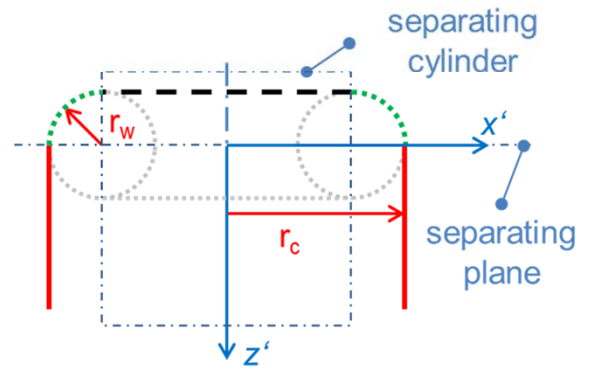
## - a) Micro cup (3D) -



### Separation rules



→ Optimal assignment based on approximated geometric parameters



# Results

## - a) Micro cup (3D) -



### Verification by Monte-Carlo simulation

cylinder radius  $r_c$  412  $\mu\text{m}$

torus radius  $r_w$  126  $\mu\text{m}$

No. of points

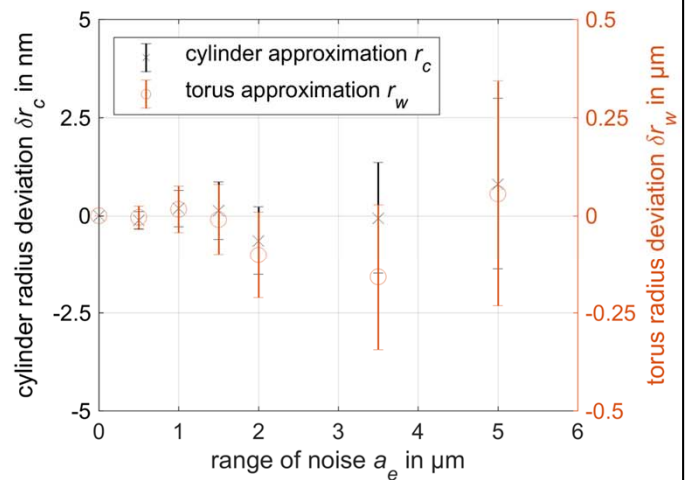
cylinder 900,000

torus 400,000

plane 366,000

} lateral  
resolution  
 $\approx 1 \mu\text{m}$

Range of noise  $[-a_e/2, a_e/2]$



→ **Precise automated evaluation of complex geometry!**

Advantages of simulation (100 repetitions with equally distributed noise):

- Referencing measurement data difficult for micro cups
- Measuring data contain deviations

Results:

- no systematic deviations / agreement with theory
- Random deviations  $< 22 \text{ nm}$  (cylinder radius)  
 $< 0.6 a_e$  (torus radius)

Note: Error bars are standard errors not enhanced uncertainties.

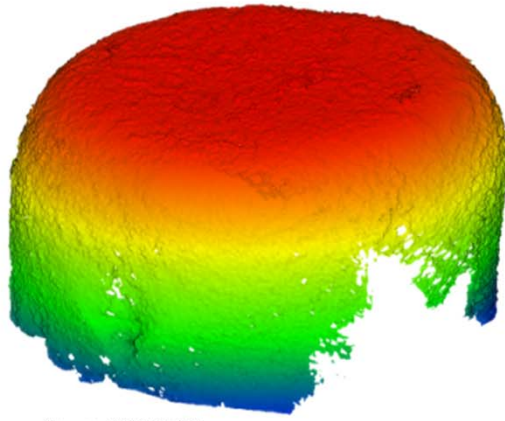
# Results

## - a) Micro cup (3D) -



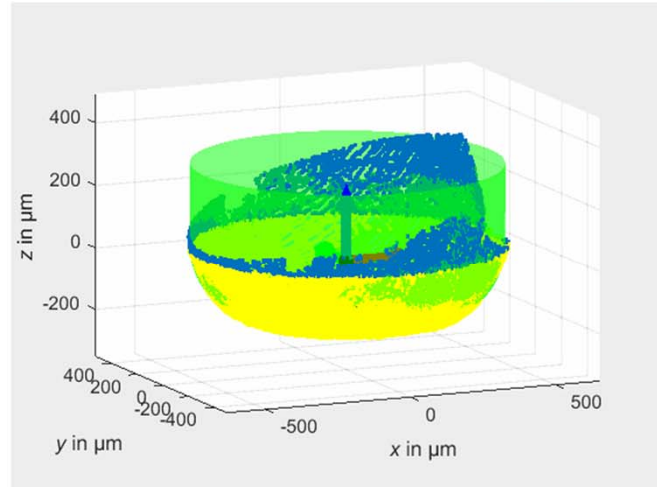
### Validation with experimental data

Surface acquired by digital holography:



Source: BIAS GmbH

500  $\mu\text{m}$

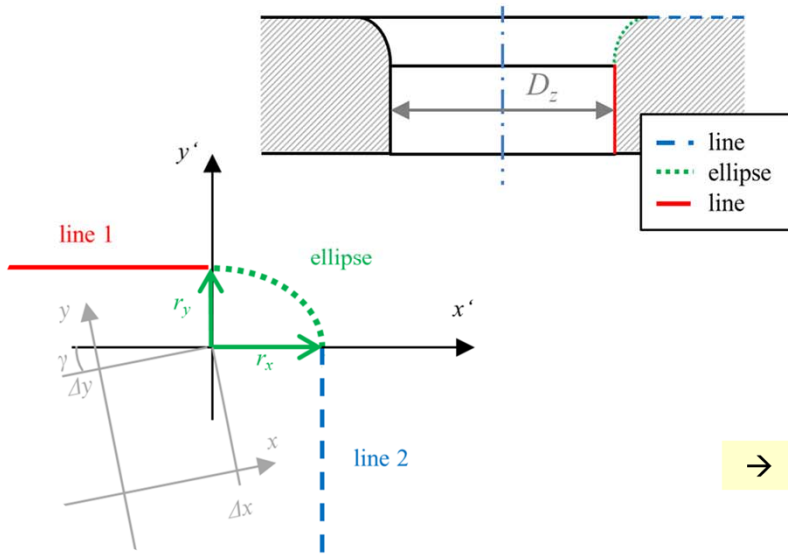


# Results

## - b) Drawing die (ellipse) -



### Geometric model



transformation parameters:

$$\vec{a}_p = [\Delta x, \Delta y, \Delta z, \varphi_x, \varphi_y]$$

shape parameters:

$$\vec{a}_g = [r_x, r_y]$$

→ No analytic solution for  $d_i$ .

# Root point iteration

- Calculation of orthogonal distance -



**Circle**

$$d_i = f(\vec{a}_p, \vec{a}_g) = \sqrt{(x_i - X_M)^2 + (y_i - Y_M)^2} - R$$

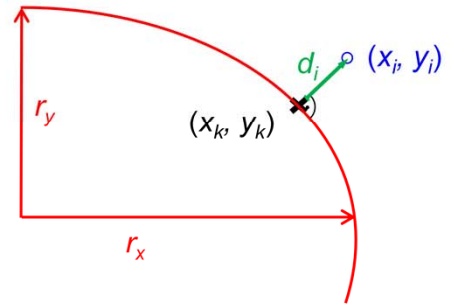
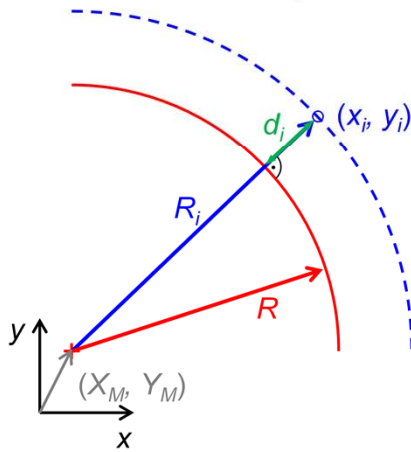
analytic

← solution →

**Ellipse**

$$d_i = f(\vec{a}_p, \vec{a}_g, \vec{x}_k)$$

numeric



→ **Enhanced holistic approximation with root point iteration.**

# Results

## - b) Drawing die (ellipse) -



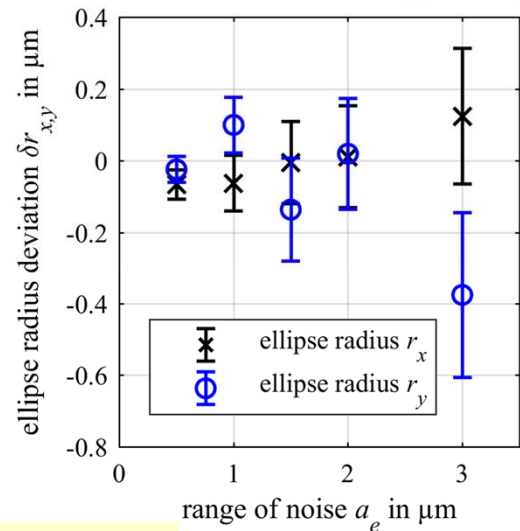
### Verification by Monte-Carlo simulation

ellipse half axis  $r_x$  373  $\mu\text{m}$

ellipse half axis  $r_y$  324  $\mu\text{m}$

No. of points  
per element 55

Range of noise  $[-a_e/2, a_e/2]$



→ **Precise automated evaluation of complex geometry!**

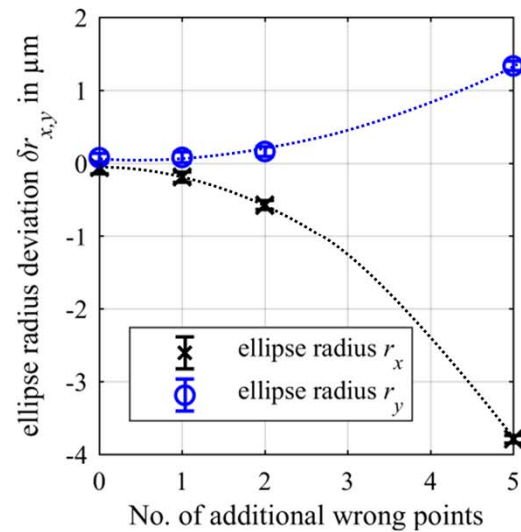
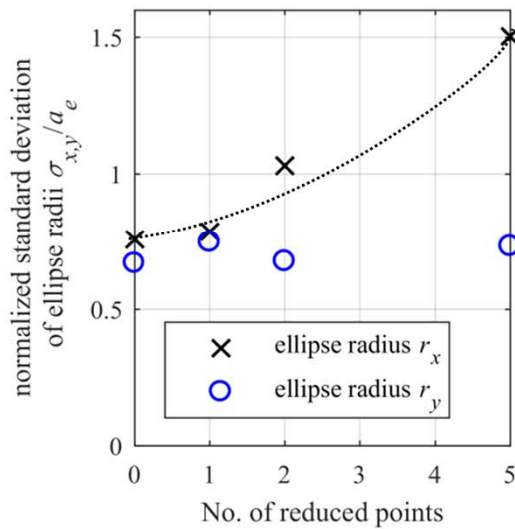
Analysis of variances (ANOVA): The prerequisites of independent evaluations and Gaussian distributed random variables are fulfilled, but a Levene's test revealed that equal variances could not be assumed. Therefore, a Welch ANOVA was used to analyze systematic deviations. The Welch test delivered  $F_{rx}(4; 229) = 0.34$  and  $F_{ry}(4; 222) = 1.35$ . Both values are below the critical values  $F_{crit}(0.05; 4; 229) = F_{crit}(0.05; 4; 222) = 2.41$ . Thus, with a probability of error of 5 %, it can be assumed that no systematic influence within the HA leads to significant deviations of both approximated radii.

# Results

## - b) Drawing die (ellipse) -



### Influence of non-optimal partitioning

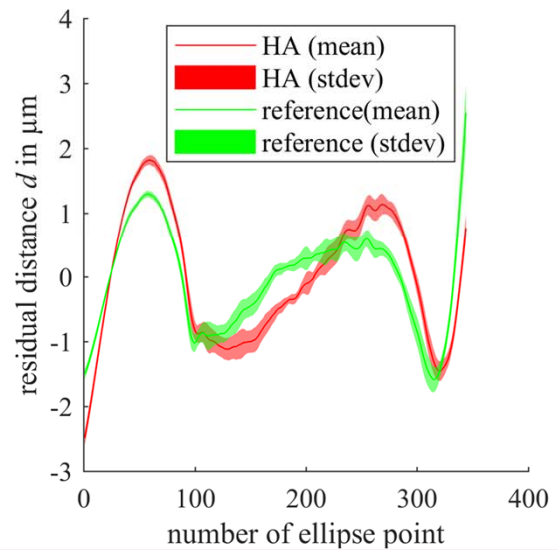
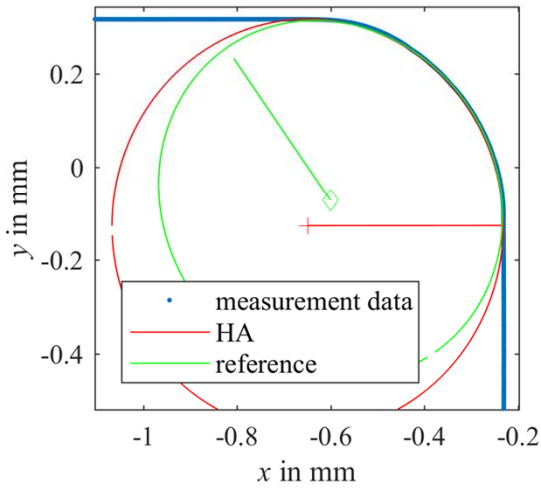


Evaluation with less than the maximum number of possible points increases the normalized standard deviation for  $r_x$  (increased random deviation).  
Evaluation with additional wrong points (of the lines) leads to a systematic error (deviation larger than standard error).

# Results

## - b) Drawing die (ellipse) -

### Validation with experimental data

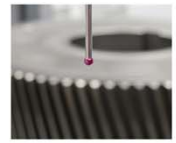


The ellipse results are not expected to be better than evaluated with commercial software (after manual partitining).  
Differences due to different degrees of freedom.  
Direct comparison with nominal data only possible with HA results!

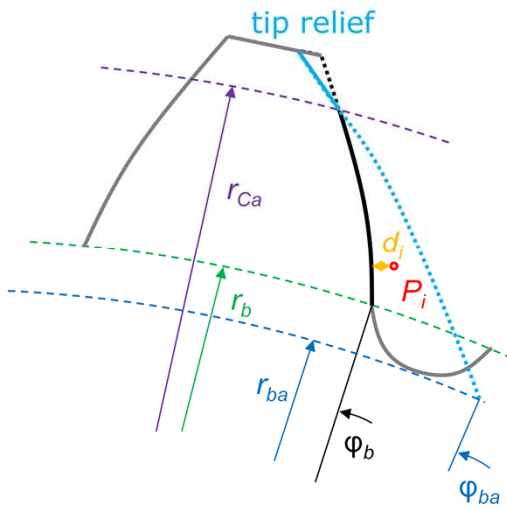


# Results

- c) Gear / involute -



## Geometric model



$$d_i = f(\vec{a}_p, \vec{a}_g)$$

transformation parameters:

$$\vec{a}_p = [\Delta x, \Delta y, \Delta z, \varphi_x, \varphi_y, \varphi_b]$$

shape parameters:

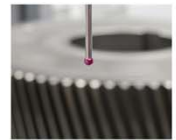
$$\vec{a}_g = [r_b, r_{ba}, r_{Ca}]$$



Identification of  
unknown gear parameters

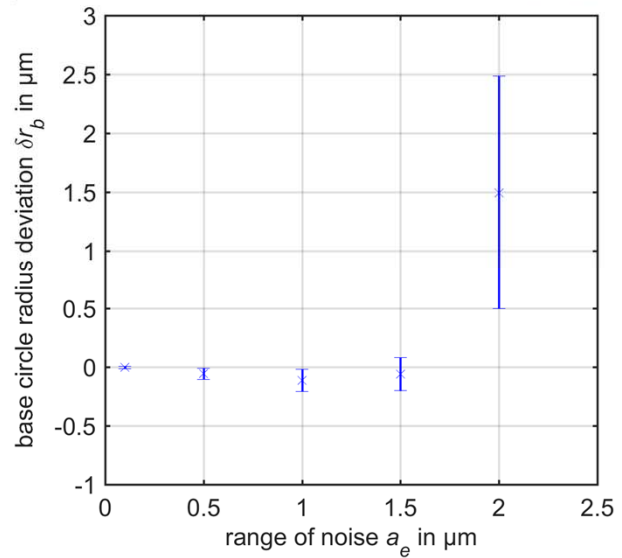
# Results

## - c) Gear / involute -



### Verification by Monte-Carlo simulation

Normal module $m_n$	5 mm
Pressure angle $\alpha$	20°
Number of teeth $n$	21
Helix angle $\beta$	0°
Pressure angle tip relief $\alpha_{Ca}$	30°
No. of points / profile	101
Range of noise	$[-a_e/2, a_e/2]$



→ **Precise automated evaluation of complex geometry!**

# Summary & outlook

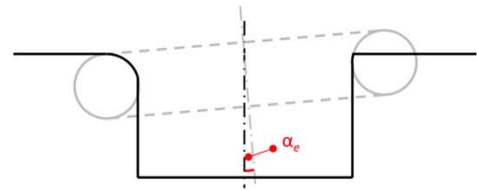
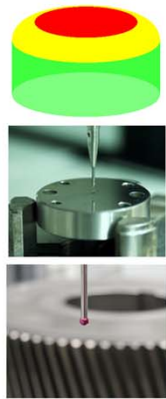
## Automated evaluation of geometric features in measurement data of combined geometric elements

### Holistic approximation (HA):

- Automated: Holistic approximation applied for various applications
- Precise: Uncertainties  $< 1 \mu\text{m}$
- Complexity: Root point iteration enables evaluation of higher order geometric elements

### Outlook

- outlier detection
- sensitivity to higher number of degrees of freedom
- weighted approximation



# Contact & acknowledgment

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