Two Uncertainty Models for Spherical Instrument Measurements

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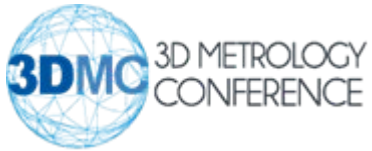
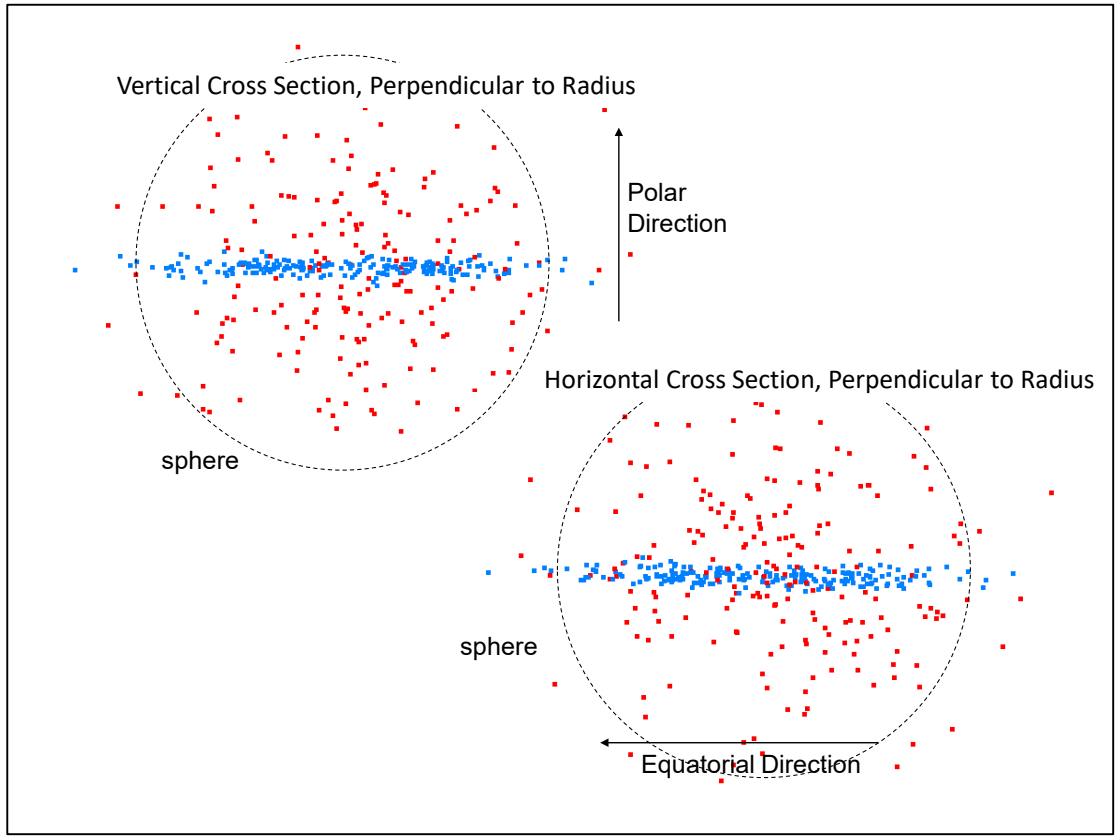
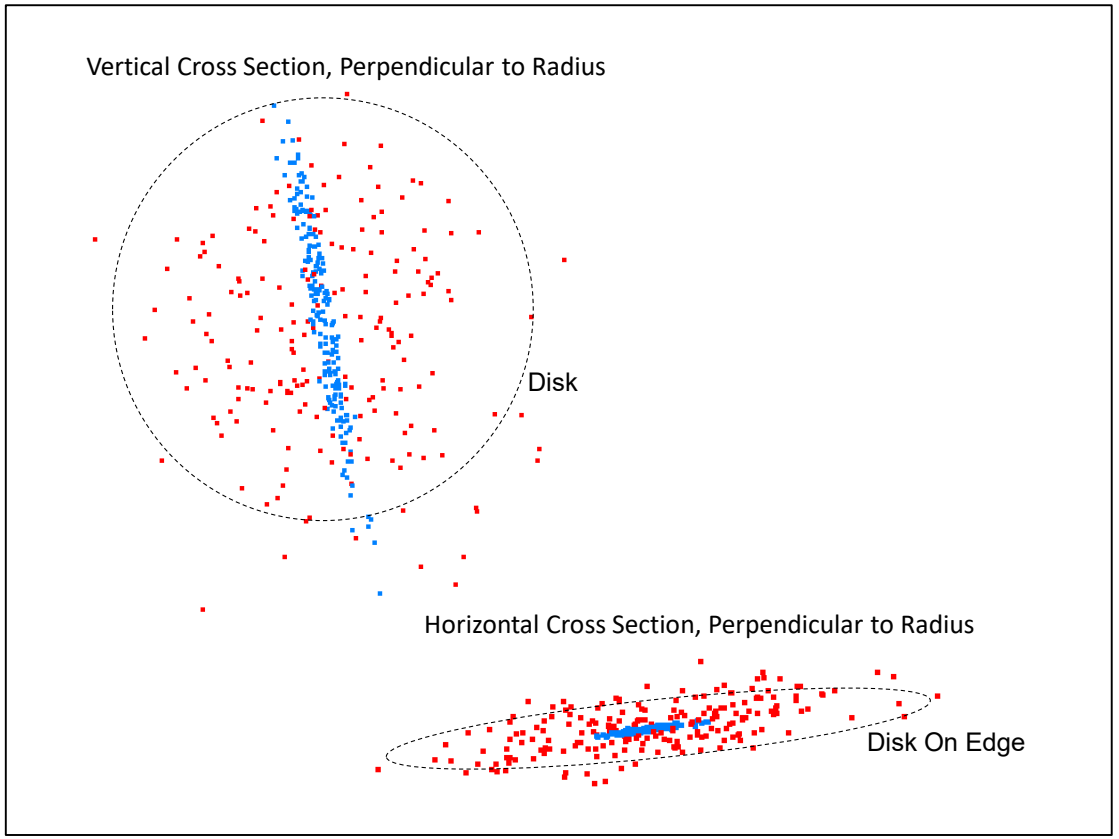
3D Metrology Conference – 3DMC



2018 Hamburg GERMANY

Background


Vertical Cross Section, Perpendicular to Radius
Polar
Direction
sphere
Horizontal Cross Section, Perpendicular to Radius
sphere
Equatorial Direction

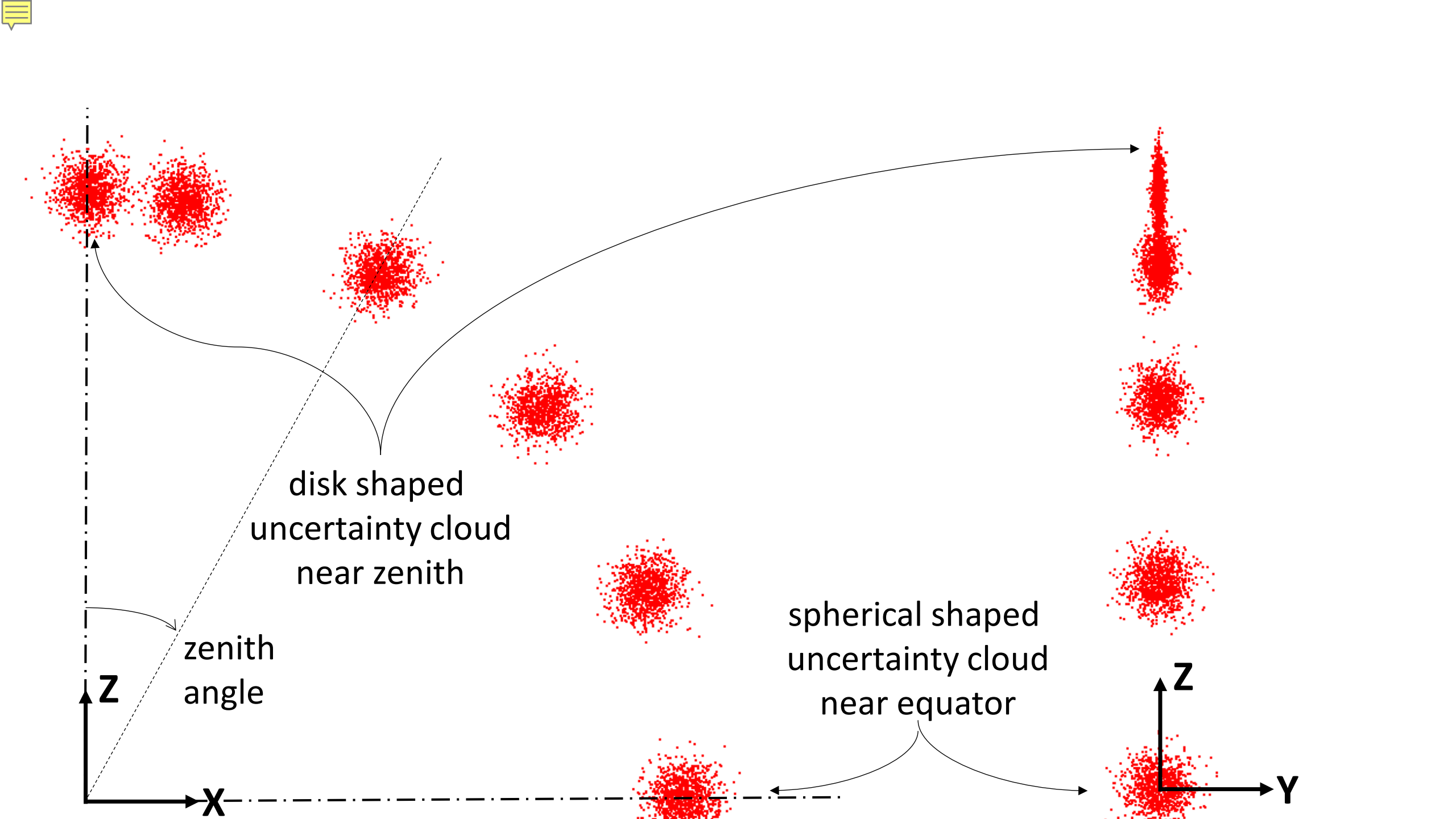

Vertical Cross Section, Perpendicular to Radius
Disk
Horizontal Cross Section, Perpendicular to Radius
Disk On Edge

Today

Cerro Pachón, Chile 2647 m (8684 ft)

Monte Carlo Uncertainty

- Starting with the Spherical to Cartesian transformation, iteratively add errors Δr, $\Delta\theta$ and $\Delta\varphi$ to the range and angles
  - $\Delta x(r,\theta,\varphi) = (r+\Delta r)\cos(\theta+\Delta\theta)\sin(\varphi+\Delta\varphi)$
  - $\Delta y(r,\theta,\varphi) = (r+\Delta r)\sin(\theta+\Delta\theta)\sin(\varphi+\Delta\varphi)$
  - $\Delta z(r,\varphi) = (r+\Delta r)\cos(\varphi+\Delta\varphi)$


disk shaped
uncertainty cloud
near zenith
zenith
angle
Z
X
spherical shaped
uncertainty cloud
near equator
Z
Y

Linear Propagation of Errors

- $U_x(r,\theta,\varphi) = \sqrt{D_{\Delta r}x(r,\theta,\varphi)^2 \cdot \Delta r^2 + D_{\Delta\theta}x(r,\theta,\varphi)^2 \cdot \Delta\theta^2 + D_{\Delta\varphi}x(r,\theta,\varphi)^2 \cdot \Delta\varphi^2}$
- $U_y(r,\theta,\varphi) = \sqrt{D_{\Delta r}y(r,\theta,\varphi)^2 \cdot \Delta r^2 + D_{\Delta\theta}y(r,\theta,\varphi)^2 \cdot \Delta\theta^2 + D_{\Delta\varphi}y(r,\theta,\varphi)^2 \cdot \Delta\varphi^2}$
- $U_z(r,\varphi) = \sqrt{D_{\Delta r}z(r,\varphi)^2 \cdot \Delta r^2 + D_{\Delta\varphi}z(r,\varphi)^2 \cdot \Delta\varphi^2}$

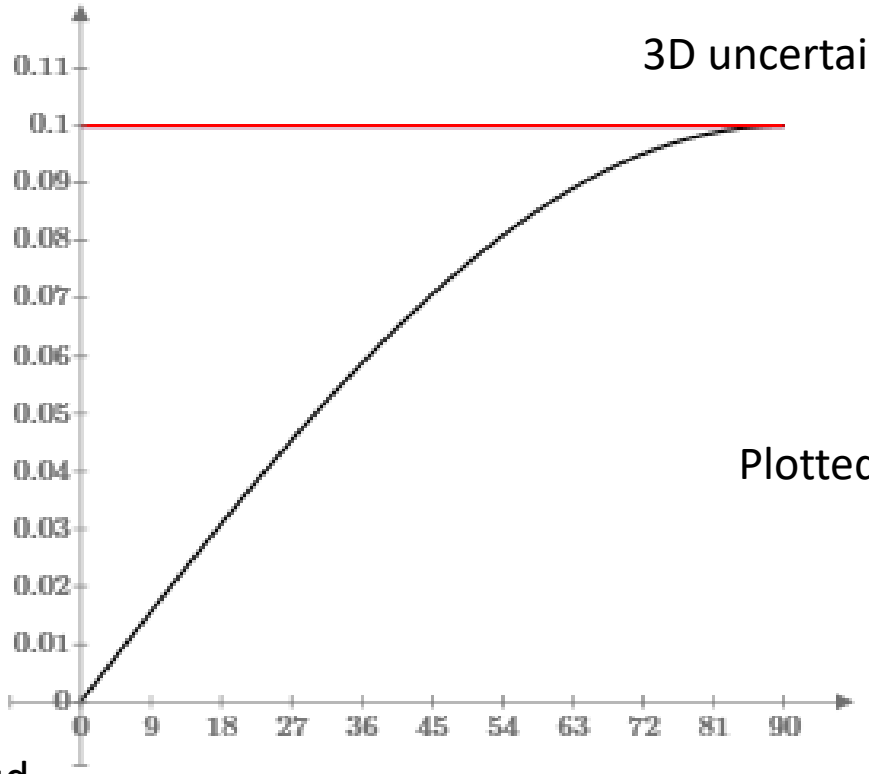
Linear Propagation of Errors

3D uncertainty cloud

$U_x(1,0,\varphi)$

$U_y(1,0,\varphi)$

$U_z(1,\varphi)$



Plotted in the X-Z plane

2D uncertainty cloud

Discussion

- Intuitively, the flattening at the poles made no sense
- Sighting through a theodolite telescope, visually there is lateral variation of the same order at all vertical angles: the field-of-view (FOV) does not squeeze into a plane
- The same is true for spherical instrument such as a tracking interferometer: both the receiver optics and quadrant detector in the tracker feedback loop have lateral variation at all angles.

- Something is amiss with the spherical to cartesian model

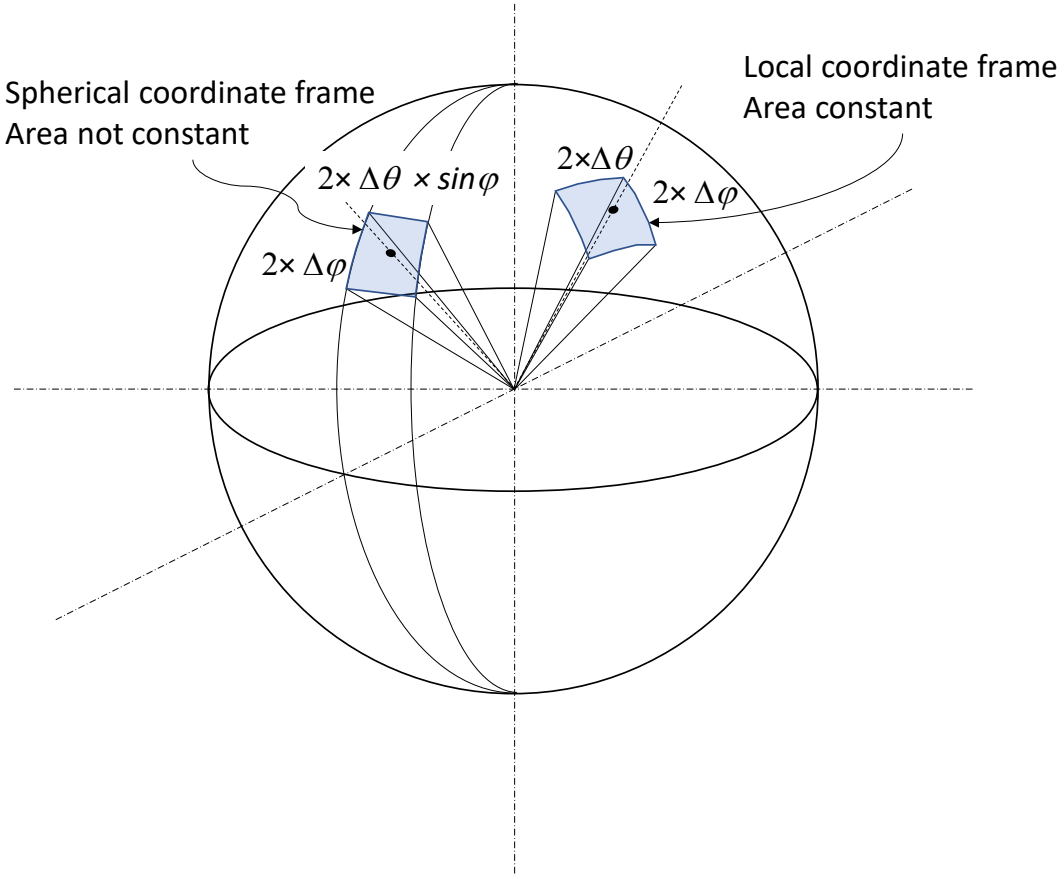
Instrument Convolution

- $m(t) = x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\lambda) \cdot \delta(t-\lambda) d\lambda = x(t)$

- Tape Deck example
  - $m(t) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\lambda) d\lambda = \int_{-\infty}^{t+\frac{T}{2}} x(\lambda) d\lambda - \int_{-\infty}^{t-\frac{T}{2}} x(\lambda) d\lambda$
  - $m(t) = x(t) * \left(u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right) \right)$

- Spherical Instrument (e.g. Telescope & Tracker) example
  - $m(\theta, \varphi) = \int_{\theta-\Delta\theta}^{\theta+\Delta\theta} \int_{\varphi-\Delta\varphi}^{\varphi+\Delta\varphi} F(1, \lambda, \eta) d\eta d\lambda$

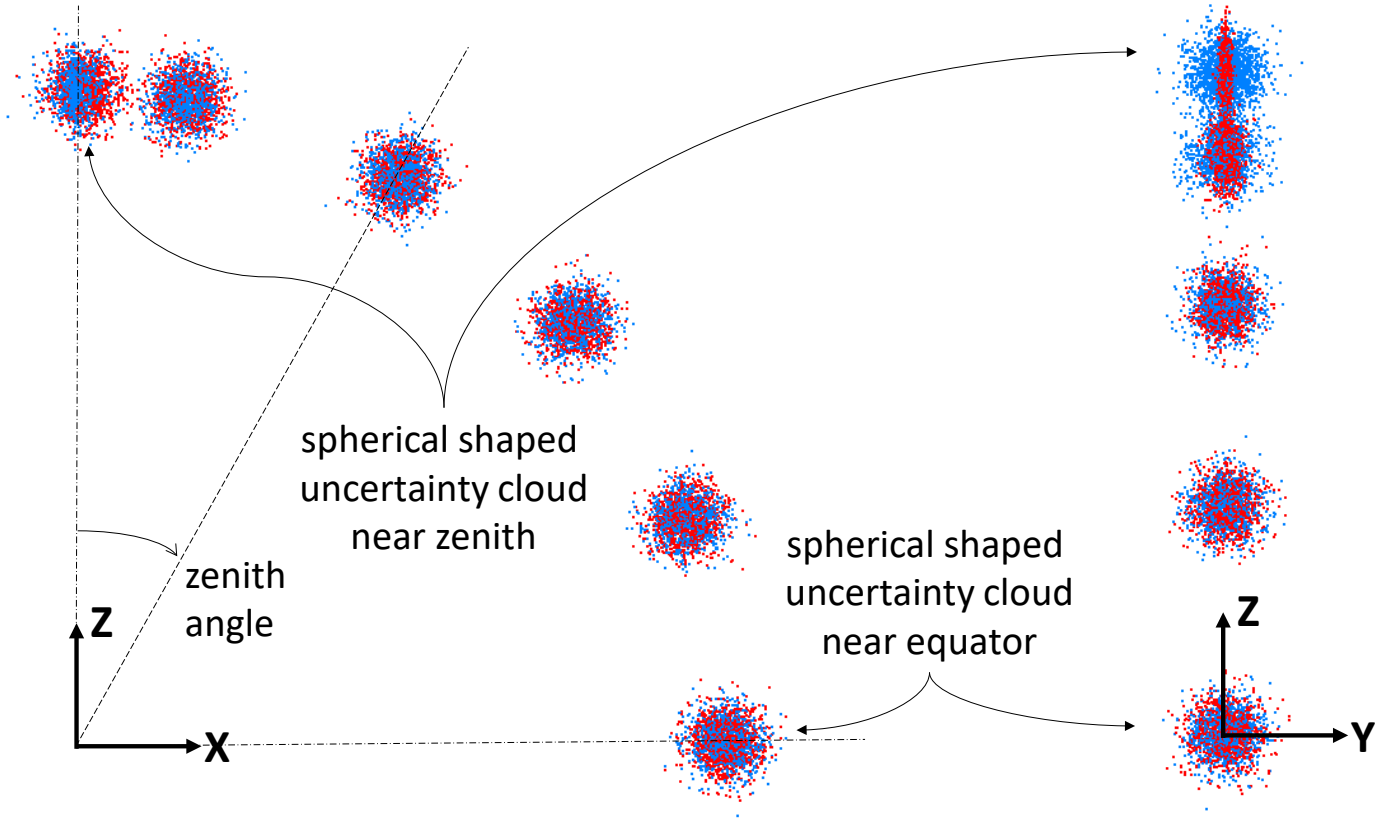
Uncertainty Regions with Constant Area



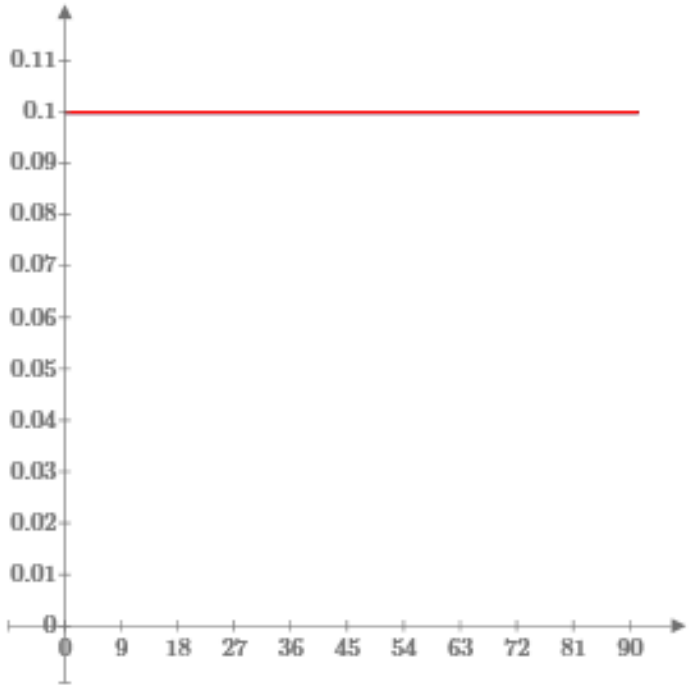
Vector Field Solution

- Measurement Vector (on-LOS) + Uncertainty Field (off-LOS)
- $$\Delta x(r,\theta,\varphi,\Delta r,\Delta\theta,\Delta\varphi) := r\cdot\sin(\varphi)\cdot\cos(\theta) \\ +\Delta r\cdot\sin(\varphi)\cdot\cos(\theta)-r\cdot\Delta\varphi\cdot\cos(\varphi)\cdot\cos(\theta)-r\cdot\Delta\theta\cdot\sin(\theta)$$
- $$\Delta y(r,\theta,\varphi,\Delta r,\Delta\theta,\Delta\varphi) := r\cdot\sin(\varphi)\cdot\sin(\theta) \\ +\Delta r\cdot\sin(\varphi)\cdot\sin(\theta)-r\cdot\Delta\varphi\cdot\cos(\varphi)\cdot\sin(\theta)+r\cdot\Delta\theta\cdot\cos(\theta)$$
- $$\Delta z(r,\varphi,\Delta r,\Delta\varphi) := r\cdot\cos(\varphi) \\ +\Delta r\cdot\cos(\varphi)+r\cdot\Delta\varphi\cdot\sin(\varphi)$$

Monte Carlo Uncertainty (compared)



Linear Propagation of Errors



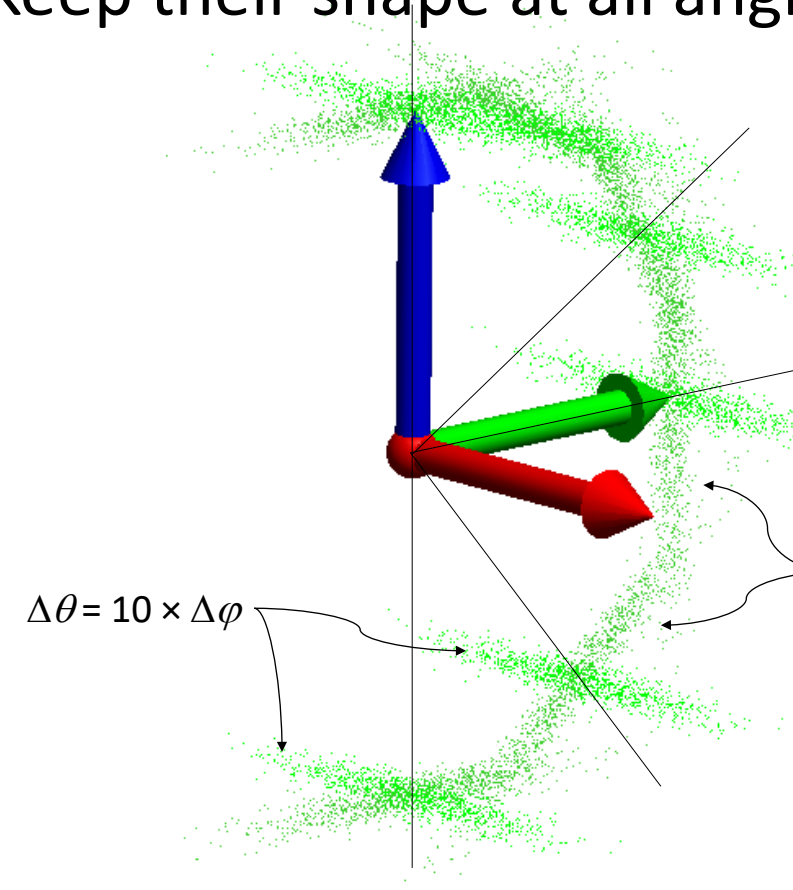
$U_x(1, 0 \cdot deg, \varphi)$

$U_y(1, 0 \cdot deg, \varphi)$

$U_z(1, \varphi)$

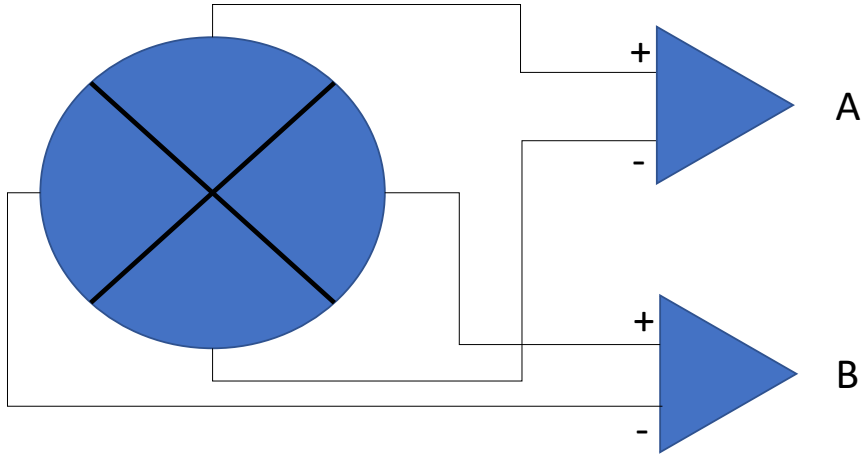
Robustness

- The point clouds keep their shape at all angles



Solution with Simple Kinematic Tracker Model using PSD Uncertainty

- Trackers PSD detects lateral error from the retro-reflector and drives servo motors, keeping the laser pointed at the retro-reflector
- The tracker PSD is the “aperture”
  - it senses the laser return
  - initiates control signals.

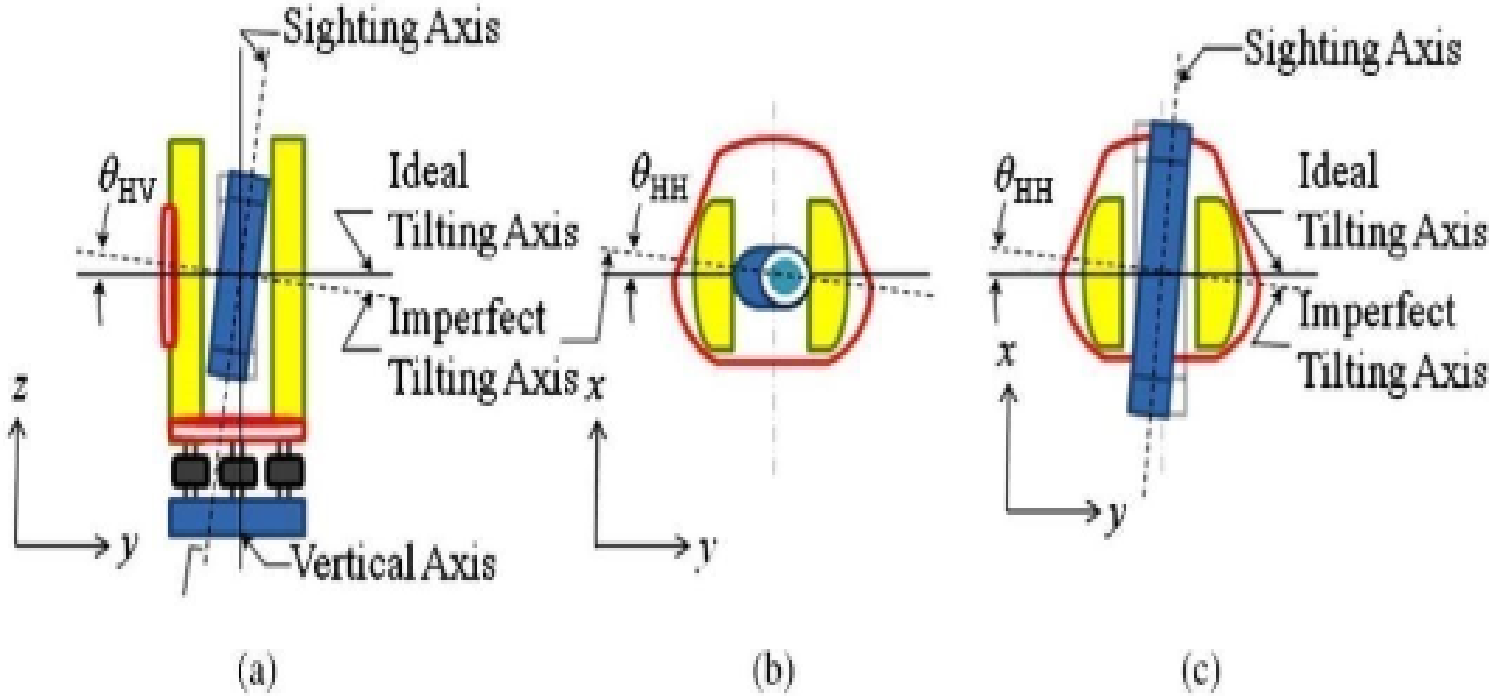


PSD and the Tracker

- The PSD does not rotate, so the azimuth angle, θ, keeps the A and B signals aligned with the alidade
  - $\begin{vmatrix}\Delta u\\ \Delta v\end{vmatrix} \approx \begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix} \times \begin{vmatrix}A\\ B\end{vmatrix}$
- Examining the values Δu and Δv with the tracker is pointing at the equator shows:
  - errors in the horizontal angle are proportional to Δu
  - errors in the zenith angle are proportional to Δv
- However, at zenith:
  - errors in the horizontal angle are <u>not</u> proportional to Δu
  - errors in the zenith angle remain proportional to Δv

Standing Axis Errors

- Two types of standing axis errors
  - Tilt in the trunnion axis
  - Mis-mounting of the laser (telescope) on the trunnion axis



PSD to Tracker Errors

- At zenith the lateral motion Δu cannot translate into a horizontal angle, but instead translates to a standing axis error.
- Δu needed to be allocated between the azimuth component, $\Delta\theta$, and a standing axis component, $\Delta\xi$.
  - $\Delta\theta \approx \frac{1}{r} \cdot \Delta u \times \sin(\varphi)$
  - $\Delta\varphi \approx \frac{1}{r} \cdot \Delta v$
  - $\Delta\xi \approx \frac{1}{r} \cdot \Delta u \times \cos(\varphi)$
- Δu and Δv are preserved
  - $\Delta\theta^2 + \Delta\varphi^2 + \Delta\xi^2 = \frac{\Delta u^2}{r^2} + \frac{\Delta v^2}{r^2}$

D-H Transforms

- D-H Transforms are a method for creating kinematic equations
  - The following is a simple tracker model
- $Rz(\theta) \cdot Ry(\varphi) \cdot T(0,0,\rho)$

$$= \begin{vmatrix} \cos(\varphi)\cos(\theta) & -\sin(\theta) & \cos(\theta)\sin(\varphi) & \rho\cos(\theta)\sin(\varphi) \\ \cos(\varphi)\sin(\theta) & \cos(\theta) & \sin(\theta)\sin(\varphi) & \rho\sin(\theta)\sin(\varphi) \\ -\sin(\varphi) & 0 & \cos(\varphi) & \rho\cos(\varphi) \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

  - Note that the last column corresponds to a spherical to cartesian transform

D-H Transform

- The standing axis error in modeled by adding a tilt in X after the zenith axis and before the translation to the SMR
- $Rz(\theta) \cdot Ry(\varphi) \cdot Rx(\xi) \cdot T(0,0,\rho)$

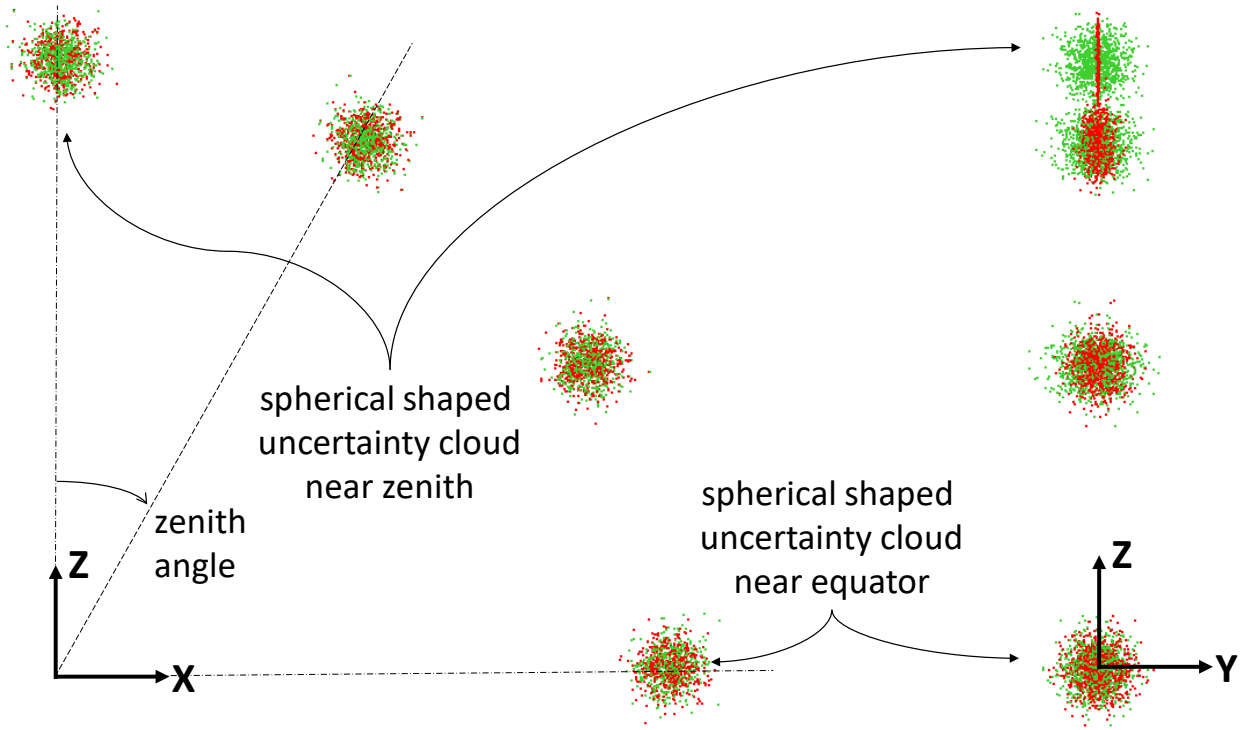
- $X(\rho,\theta,\varphi,\xi) = \rho(\cos(\theta)\sin(\varphi)\cos(\xi) + \sin(\theta)\sin(\xi))$
- $Y(\rho,\theta,\varphi,\xi) = -\rho(\sin(\theta)\sin(\varphi)\cos(\xi) + \cos(\theta)\sin(\xi))$
- $Z(\rho,\theta,\varphi,\xi) = \rho\cos(\varphi)\cos(\xi)$

Monte Carlo

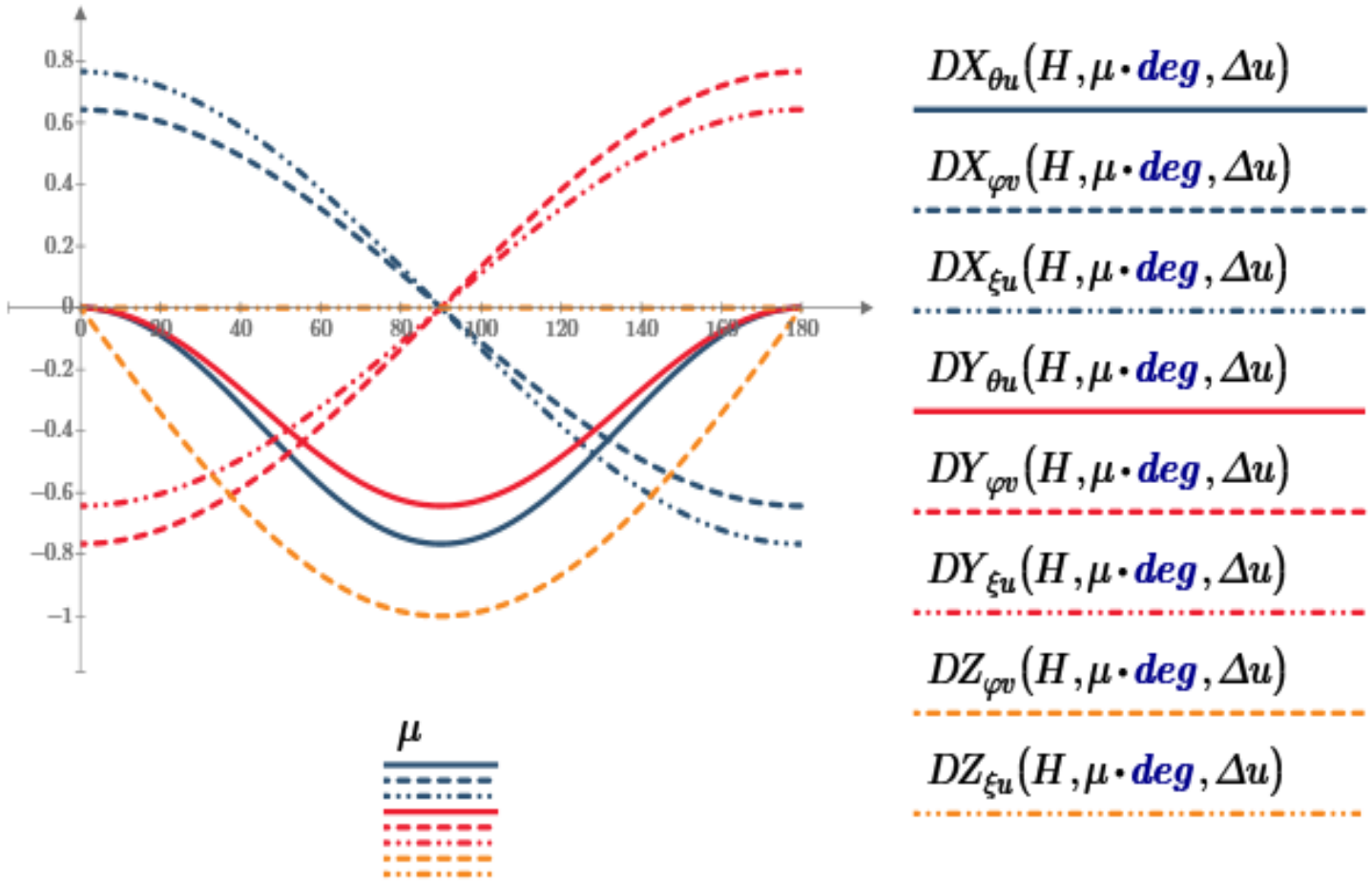
- Define the Monte Carlo errors
  - $\theta \rightarrow \theta + \Delta\theta \cdot \sin(\varphi) = \theta + \frac{\Delta u}{r}\sin(\varphi)$
  - $\varphi \rightarrow \varphi + \Delta\varphi = \varphi + \frac{\Delta v}{r}$
  - $\xi \rightarrow \Delta\theta \cos(\varphi) = \frac{\Delta u}{r}\cos(\varphi)$
  - $r \rightarrow r + \Delta r$

Monte Carlo

- Point clouds for 1000 trials

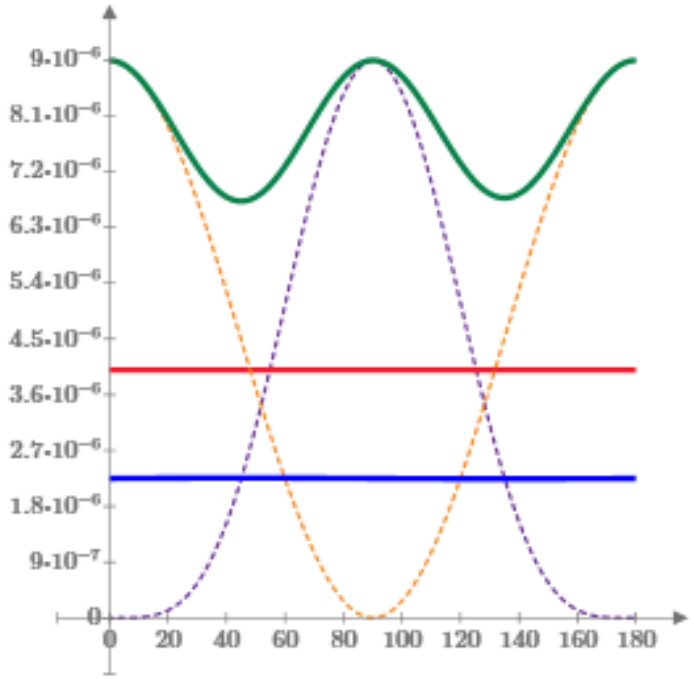


Linear Propagation of Errors



Linear Propagation of Errors

- Squaring and grouping derivative terms, and multiplying by the square of the uncertainties



$$D2_{\theta}(H,\mu\cdot deg,\Delta u)\cdot\Delta u^2$$

$$D2_{\varphi}(H,\mu\cdot deg,\Delta u)\cdot\Delta v^2$$

$$D2_{\xi}(H,\mu\cdot deg,\Delta u)\cdot\Delta u^2$$

$$D\theta\xi 2(H,\mu\cdot deg,\Delta u)\cdot\Delta u^2$$

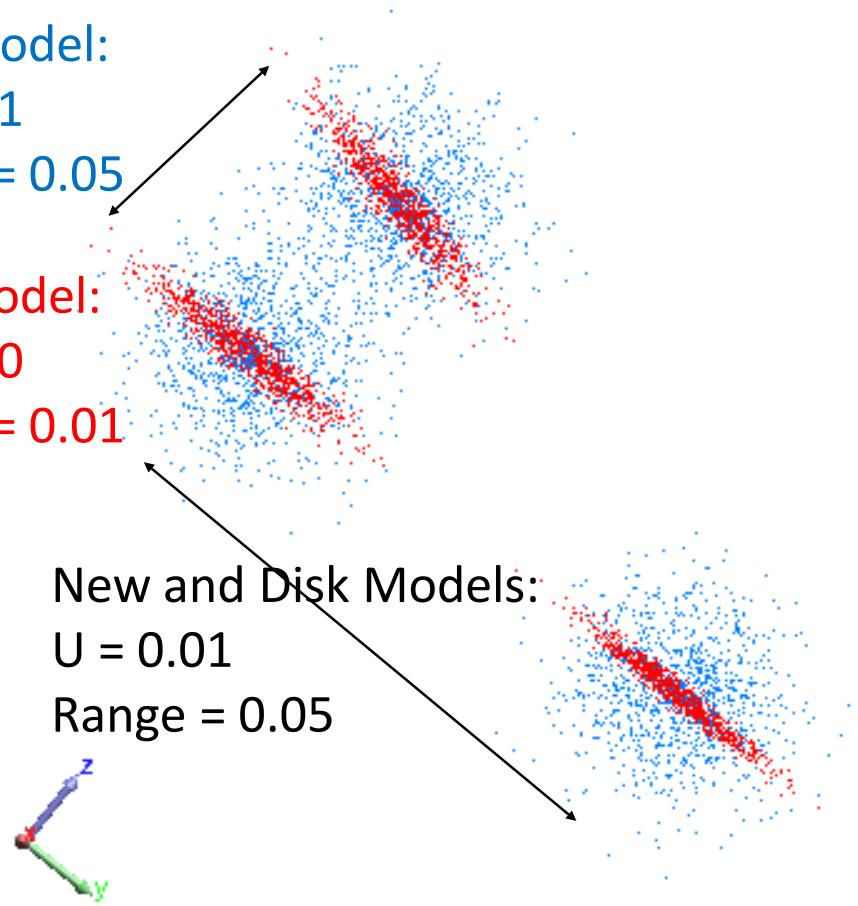
$$D2_{\rho}(H,\mu\cdot deg,\Delta u)\cdot\Delta\rho^2$$

Point-to-Point Comparison

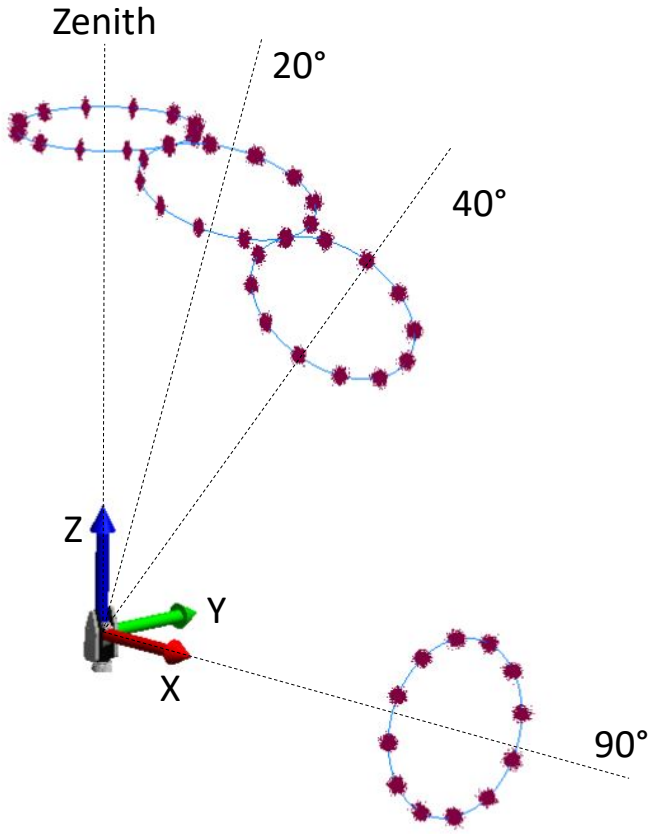
New Model:
U = 0.01
Range = 0.05

Disk Model:
U = 0.00
Range = 0.01

New and Disk Models:
U = 0.01
Range = 0.05



Points-to-Objects Comparison



Points-to-Object Comparison

- Solutions for Spherical to Cartesian and New Models are the same
- Standard Deviations are different
  - New model has larger Standard Deviation in some cases
- The analysis of disk and spherical shaped uncertainty clouds using points-to-circles evaluation in above figure is specific to that figure.

	unconstrained Radius			
	disk point cloud		*spherical point cloud*	
	value (mm)	*Stdev (mm)*	*value (mm)*	*Stdev (mm)*
	zenith		*zenith*	
X	0.0000	0.0055	0.0000	0.0055
Y	0.0000	0.0056	0.0000	0.0056
Z	3939.2310	0.0111	3939.2310	0.0111
Radius	694.5930	0.0038	694.5930	0.0038
	20 deg		*20 deg*	
X	1347.2964	0.0061	1347.2964	0.0066
Y	0.0000	0.0041	0.0001	0.0056
Z	3701.6663	0.0109	3701.6663	0.0109
Radius	694.5930	0.0033	694.5929	0.0041
	40 deg		*40 deg*	
X	2532.0890	0.0080	2532.0890	0.0082
Y	0.0000	0.0045	0.0000	0.0053
Z	3017.6260	0.0092	3017.6260	0.0093
Radius	694.5930	0.0034	694.5930	0.0040
	90 deg		*90 deg*	
X	3939.2310	0.0111	3939.2310	0.0111
Y	0.0000	0.0056	0.0000	0.0055
Z	0.0000	0.0056	0.0000	0.0056
Radius	694.5930	0.0039	694.5930	0.0040

Conclusion

- New model for point cloud uncertainty for spherical measurement was shown
  - On-LOS and off-LOS rays were treated differently
  - Monte Carlo uncertainty clouds stayed constant at all horizontal and zenith angles
  - Linear error propagation also stays constant at all angles
- On-LOS measurement values match other models
- Off-LOS uncertainties can differ from other models
  - Uncertainties are task specific
  - Compared to Monte Carlo methods the new model may converge more slowly than spherical to cartesian model

Acknowledgements

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- Thanks to the Large Synoptic Survey Telescope for putting me back on track and looking at this uncertainty issue again.

References

- Hammersley, J.M., Handscomb, D.C., “Monte Carlo Methods”, Methuen & Co. Ltd, London, 1964.
- “Evaluation of measurement data — Supplement 2 to the “Guide to the expression of uncertainty in measurement” — Extension to any number of output quantities”, JCGM 102:2011
- Dorsey-Palmateer, J.W., “Spinning Mirror Laser Radar Kinematic Errors”, Large Volume Metrology Conference, 2012.
- Hayden, J., Khreishi, M., Hadjimichael, T., and R. Ohl, R., “Monte Carlo Method for Uncertainty Propagation in JWST Metrology Databases”, Coordinate Metrology Society Conference, 2014.
- Calkins, J., Sandwith, S., “Integrating Certified Lengths to Strengthen Metrology Network Uncertainty”, Coordinate Metrology Society Conference, 2007.
- Brown, E.B., “Modern Optics”, Reinhold Publishing Corp, 1965.
- McGillem, C.D., Cooper, G.R., "Continuous and Discrete Signal and System Analysis", Holt, Rinehard, and Winston, Inc., 1974.
- Kam C. Lau, Robert J. Hocken, "Three and five axis laser tracking systems ", US Patent 4714339, December 22, 1987.
- Lawrence B. Brown, David N. Wells, J. Bradford Merry, "Tracking laser interferometer", US Patent 4790651, December 13, 1988.
- Paul, R.P., "Robot Manipulators", MIT Press, 1981.
- Shui, R., Kang, S., Han, J.H., Hsieh, M., "Modeling Systematic Errors for the Angle Measurement in a Virtual Surveying Instrument", J. Surveying Eng., Aug 2011.
- Palmateer, J.W., “Those #%&!$ Corner Cubes”, Coordinate Metrology Society Conference, 2002.
- Kreyszig, E., “Advanced Engineering Mathematics 10th Edition”, J. Wiley and Sons, 2011.